Experimental and Analytical Study of Adaptive Structures using Eigenstrain Techniques

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ABSTRACT

The dynamic response of an adaptive beam containing embedded mini-devices (sensors and actuators) is investigated analytically and experimentally in this paper. The dynamic model is based on Hamilton's variational principle and the mechanical interactions between the beam and the devices are modeled using Eshelby's equivalent-inclusion method. The dynamic model is verified experimentally, using a cantilever beam made of ALPLEX plastic as host material and piezoelectric devices (PZT-5H) as active mini-devices for sensing and actuation. The experimental setup is outlined and the analytical results are compared with the experimental ones. The capability of mini-actuators to change the dynamic behavior of the adaptive beam is explored for the adaptive stiffening case.

KeyWords: adaptive structures, eigenstrain techniques, adaptive stiffening

2. INTRODUCTION

Adaptive materials and intelligent systems have been the focus of researchers for quite sometime. Adaptive structures have been used in the area of active vibration control by many researchers, see for example [Burke and Hubbard, 1987], [Tzou and Tseng, 1990], [Ha et al, 1992], [Alghamdi and Dasgupta, 1993a]. Piezoelectric materials have attracted significant attention for their potential application as sensors and actuators for controlling the response of structures. Distributions of sensors, actuators, and data processing capability are used to modify, tune, and control the response of adaptive structures to sensed stimuli.

Numerous studies have modeled the interactions between devices and hosts in smart structures. The response of adaptive structures with laminated assemblies of piezoelectric wafers/films in composite beams have been analyzed by laminate analysis [Crawley and Lazarus, 1991], simple beam models [Bailey and Hubbard, 1985], pin force models [Crawley and de Luis, 1987], large deformation beam theory [Im and Atluri, 1989], and one-dimensional eigen-function approximations [Lin and Rogers, 1992]. Variational methods have also been developed for solving the coupled boundary value problems in adaptive structures. These include a strain energy method [Wang and Rogers, 1991], finite element methods [Allik and Hughes, 1970], and Rayleigh Ritz methods [Hagood et al, 1990].

Most present-day adaptive structural systems consist of relatively large surface-mounted "active" elements which can cause reliability problems due to high stress concentrations, poor interfacial bonding, change in the boundary conditions, etc. These limitations can be partially overcome by using mini-devices with less obtrusivity, and embedding them throughout the host to achieve adequate control authority. However, embedding produces three-dimensional stress interactions that are more difficult to model than those arising in surface-mounted devices.

Eshelby's technique [Eshelby, 1957] offers a convenient method to model the mechanical interactions in systems with embedded mini-devices [Dasgupta and Alghamdi, 1992]. Eshelby's

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equivalent-inclusion method has been previously used by the authors to model the elastic interaction between actuators/sensors and the host, by using appropriate Green's functions. The authors have developed analytical solutions for embedded mini-devices using eigenstrain techniques, see for example [Dasgupta and Alghamdi, 1992], [Dasgupta and Alghamdi, 1993], and [Alghamdi and Dasgupta, 1994].

The focus of the present paper is on the behavior of a beam with embedded mini-devices. The devices are small compared to the characteristic dimensions of the host and the maximum volume fraction of the devices is below 2 %. The equation of motion of the system is derived using a generalized Hamilton's principle. The change in the natural frequency of the structure due to harmonic excitation of the actuator is examined analytically.

Experimental verification of the analytical model is presented. Adaptivity of the beam is illustrated through adaptive stiffening of the cantilever beam for position feedback control. The authors have developed details of the analytical model in a previous paper [Alghamdi and Dasgupta, 1993b] for voltage driven electrodes, and have presented experimental data of the adaptive beam in another paper [Alghamdi and Dasgupta, 1994]. In this paper the model for charge-driven electrodes is developed, and comparison between the analytical model and the experimental results is illustrated.

3. ANALYSIS

As shown schematically in Figure (1), two rows of uniformly spaced micro-devices are embedded in the beam symmetrically about the neutral plane of the beam. One row is used as sensors and the other as actuators. As the beam flexes, sensor outputs are used in a position-feedback circuit to actuate the corresponding active devices (actuators) in the opposite row. The result is a stiffening of the beam and an accompanying increase in the natural frequency ω, if all losses in the system are ignored.

In this study, each micro-device is approximated to act like elastic heterogeneities embedded in a large host structure. Host and devices materials are approximated to be linear and mechanically isotropic. All material properties are listed in Table (1).

The variational principle is a generalized form of Hamilton's principle, and may be written as [Tiersten, 1967];

$$\delta \int_{t_0}^{t} (L + W) \, dt = 0 \tag{1}$$

where the Lagrangian functional L is the difference between the kinetic energy and the electric enthalpy. The work term, W, includes the work done by electrical charges imposed at the actuator electrodes

The linear isothermal constitutive relation for piezoelectric material is given as:

$$\begin{aligned} \sigma_{ij} &= C^{E}_{ijkl} \, \epsilon_{kl} - e_{kij} \, E_k \\ D_i &= e_{ikl} \, \epsilon_{kl} + \varepsilon^{s}_{ij} \, E_j \end{aligned} \tag{2}$$

where σ_{ij} is the stress tensor, ϵ_{kl} is the strain tensor, E_i is the electrical field vector, D_i is the electrical displacement vector, C^{E}_{ijkl} is the mechanical stiffness tensor at constant electrical field, e_{ijk} is the piezoelectric stress coupling tensor, and ε is the dielectric tensor at constant strain.

Thus, using the definition of electric enthalpy and substituting for the work term and taking the variation one can get [Alghamdi and Dasgupta, 1993c]:

$$\int_{t_o}^{t} [\int_{v} \ddot{u}_i \rho \delta u_i \, dv + \int_{v} \epsilon_{ij} C_{ijkl} \delta \epsilon_{kl} \, dv - \int_{\Omega} \epsilon_{ij} e_{kij} \delta E_k \, dv - \int_{\Omega} \delta \epsilon_{ij} \, e_{kij} \, E_k \, dv - \int_{\Omega} E_i \, \varepsilon_{ij} \, \delta E_j \, dv - \int_{s} Q \delta V \, dA \,] \, dt = 0 \quad (3)$$

where u_i is the displacement vector, ρ is the density, v is the volume of the beam, Ω is the volume of the devices, Q is the specified surface charge density, V is the electric potential, and A is the electrode surface area of the devices. The integration domain in the above equation covers host, sensors and actuators, and the structural effect of sensors and actuators on the structure is considered. Also, the direct piezoelectric effect at sensors and actuators and the converse effect at actuators are considered.

Eshelby's classical equivalent-inclusion technique is applied to obtain the elastic interaction fields, both in the device and in the host, under external applied loads and under internal actuation loads [Eshelby, 1957]. The internal actuation loads are treated as real eigenstrains. In the present analytical context, sensors and actuators have fictitious eigenstrain due to external far-field loads and real eigenstrain due to direct effect, while actuators have additional real eigenstrains due to the converse effect of the actuation voltage. The real eigenstrain due to the direct effect are ignored in this study.

Eshelby's method is based upon postulating an equivalent-inclusion with a fictitious eigenstrain which has the same stress field as the real heterogeneity, under both external loads and internal actuation strains. Thus, in the heterogeneity:

$$\sigma^{0}_{ij} + \sigma'_{ij} = C^{D}_{ijkl} \, (\epsilon^{0}_{kl} + \epsilon'_{kl} - \epsilon^{r}_{kl}) = C^{H}_{ijkl} \, (\epsilon^{0}_{kl} + \epsilon'_{kl} - \epsilon^{*}_{kl}) \quad (4)$$

where $\epsilon^{*}_{ij} = \epsilon^{r}_{ij} + \epsilon^{f}_{ij}$. Superscripts D and H on the stiffness indicate the device and the host, respectively, and superscripts 0, / , r, f and * on the stress and strain terms indicate applied far-field value, disturbance due to the presence of the heterogeneity, real actuation eigenstrains, fictitious eigenstrains due to external loading, and total eigenstrains, respectively. Details of the eigenstrain method can be found in [Dasgupta and Alghamdi, 1992].

The total strain inside the devices can be calculated using eigenstrain techniques [Dasgupta and Alghamdi, 1994] as:

$$\epsilon_{ij} = \epsilon^{o}_{ij} + \epsilon'_{ij} = \epsilon^{o}_{ij} + S_{ijkl} \epsilon^{*}_{kl} \quad (5)$$

where S_{ijkl} is Eshelbey's fourth-order strain concentration tensor [Eshelby, 1957].

The second term in Equation (2) can be written in terms of the eigenstrain [Alghamdi and

Dasgupta, 1993c] as:

$$\begin{aligned}\int_V \epsilon_{ij} C_{ijkl} \delta\epsilon_{kl}\, dV = \\ \int_V \epsilon^o_{ij} C^H_{ijkl} \delta\epsilon^o_{kl}\, dV + \int_\Omega \epsilon^*_{ij} C^H_{ijkl} S_{klmn} \delta\epsilon^*_{mn}\, dV \\ + \int_\Omega \epsilon^o_{ij} \Delta C_{ijkl} \delta\epsilon^o_{kl}\, dV + \int_\Omega \epsilon^o_{ij} \Delta C_{ijkl} S_{klmn} \delta\epsilon^*_{mn}\, dV \\ + \int_\Omega \delta\epsilon^o_{ij} \Delta C_{ijkl} S_{klmn} \epsilon^*_{mn}\, dV + \int_\Omega \epsilon^*_{ij} S_{ijkl} \Delta C_{klmn} S_{mnpr} \delta\epsilon^*_{pr}\, dV\end{aligned} \tag{6}$$

where $\Delta C_{ijkl} = C^D_{ijkl} - C^H_{ijkl}$.

Assuming an Euler-Bernoulli beam formulation for the host cantilever, an admissible displacement function w in z-direction is assumed in accordance with the Rayleigh-Ritz technique:

$$w = [\ 1 - \cos(\frac{\pi y}{2l})\ ]\ r(t) \tag{7}$$

where r(t) is a generalized degree-of-freedom. For simplicity, only one degree-of-freedom is used in this study. This is adequate for modeling the first vibrational mode.

The corresponding voltage in the sensor is assumed to be proportional to the strain at that sensor

$$E_s = -\ z\ (\frac{\pi^2}{4l^2})\ \cos(\frac{\pi y}{2l})\ V_s(t) \tag{8}$$

where $V_s(t)$ is the generalized sensor degree-of-freedom.

Similarly, the electrical field at the actuator is assumed to be;

$$E_a = -\ z\ (\frac{\pi^2}{4l^2})\ \cos(\frac{\pi y}{2l})\ V_a(t) \tag{9}$$

where $V_a(t)$ is the generalized actuator degree-of-freedom. Substituting Equations (5) and (6) into Equation (3) and allowing arbitrary variation of r(t) and $V_a(t)$, one obtains the following set of equations for the system:

$$M\ \ddot{r}(t)\ +\ K_p\ r(t)\ +\ C_a\ V_a(t)\ =\ 0 \tag{10}$$

$$C_a\ r(t)\ +\ S_a\ V_a(t)\ =\ q_a \tag{11}$$

where M, K_p, C, and S are the mass, stiffness, electromechnical coupling, and capacitance, respectively, and q is the applied charges. Details of these terms can be found in [Dasgupta and Alghamdi, 1994]. Subscripts a and s refer to actuator and sensor, respectively. In reality there is also a structural or viscous damping term which has been ignored in Equation (10).

In a charge-controlled system, $V_s = 0$, and the charge generated at sensors is proportional to displacement r(t) and can be calculated as:

$$q_s = \int_{A_s} e_{ijk} \, \epsilon_{jk} \, r(t) \; n_i \; dA \tag{12}$$

where A_s is the area of the sensors electrodes and n_i is the unit outward normal vector. Typically Equation (11) is used to relate the actuation voltage, V_a, to deformation r(t).

In this study, we assume charge-driven system such that the charges applied to actuators are proportional to the charges generated at the sensors,

$$q_a = G \, q_s \, r(t) \tag{13}$$

where G is the gain.

For simplicity of illustration in this paper, we assume only the first vibrational mode. Substituting Equation (13) into Equation (11) and Equation (11) into Equation (10), the square of the first natural frequency can be written as:

$$\omega^2 = \frac{K_p + G \, C_a S_a^{-1} \, q_s - C_a S_a^{-1} C_a}{M} \tag{14}$$

In the following section analytical results are compared with the experimental ones for different gains G.

4. ADAPTIVE CANTILEVER BEAM

Figure (1) shows the specimen used in the test. Two rows of piezoceramic (PZT) devices are embedded within the ALPLEX beam symmetrically about the beam midplane. The volume of each device is 0.22% of the beam volume. These devices are grouped, for convenience, into four pairs, shown by the dashed boxes in Figure (1). In the first two pairs, one device acts as a sensor, and the other as an actuator. Both devices of the third pair act as actuators, and can be excited 180° out-of-phase to excite a pure bending mode for characterizing the frequency response function [Alghamdi and Dasgupta, 1994]. In the fourth pair, both devices act as sensors. Figure (2) illustrates the experimental setup. The symmetry in the locations of the sensor and actuator in each pair provides certain advantages similar to collocated sensors and actuators, for distributed control strategies.

Sensor outputs from first and second pairs are amplified using two different charge amplifiers. The output is then passed through a phase shifter and a low-pass frequency filter. A power amplifier is used to amplify the signal before it goes to the actuators. Details of the experimental setup is given in [Alghamdi, 1994].

5. RESULTS AND DISCUSSIONS

In this study only the first vibration mode is investigated. As discussed in Section 5, locating the sensors of the active pairs symmetrically with respect to the actuator locations, provides a type of distributed control. As a result, the history of the applied charges in each actuator is different, even though the gain is constant. However, at any given time the field in each actuator is proportional to its location along the y axis (see Figure (1)). This proportionality allows us to characterize the field histories in all the actuators, by just one parameter. In this study, the gain, G, is chosen as the defining parameter.

Figure (3) represents the analytical and experimental free vibration responses of the beam due to an initial manual displacement applied at the free end. The experimental time response shown represents sensor voltage at pair 4, normalized by the value at t = 0. The solid line is the analytical damped time response due to some assumed structural damping, with all devices passive (zero gain, G = 0). The viscous damping assumed in the model is based on experimental observation. The dashed line represents the experimental response with no actuators active (G = 0). It can be seen that the analytical model agrees well with the experimental results and the predicted first natural frequency is 23.35 Hz while the measured one is 24.4 Hz.

Figure (4) presents analytical and experimental responses for the adaptive beam with two active devices (n = 2) at gain G = 38. Experimental response shows stiffening accompanied by some additional damping because the applied charges tend to suppress the vibration. Damping due to position feedback was reported in the literature (see for example [Fanson et al, 1989]). It is worth mentioning at this point that the small change in the dynamic response is due to the small volume fraction of the devices which is only 0.44% for each pair. The calculated first natural frequency is 24.23 Hz and the experimental one is 25.3 Hz.

The effect of changing the gain (G) on stiffening is shown in Figure (5). The y-axis is the normalized fundamental frequency of the adaptive beam normalized with respect to the frequency of the passive beam (G = 0). The experimental values of the fundamental frequency are the corresponding frequencies of the peaks of FRF plots (measured at pair IV) obtained from the spectrum analyzer. The figure shows the system behavior for two (n = 2) active pairs of devices. It can be seen that as the gain increases, the induced strain increases, and hence the stiffening increases. The experimental values show good agreement with the analytical model predictions.

Table (II) illustrates the analytical and experimental first natural frequency of the beam for passive and active cases. In this table number of active devices are varied from 0 to 2 at constant gain (G = 38) and the analytical predictions are compared with the experimental results in terms of the percentage error. The maximum percentage error is 5.3% at n = 1.

Table (III) gives the analytical prediction and the experimental results for two active device pairs (n = 2) at different values of the gain (G). Excellent agreement is shown and the maximum error is 5% at G = 50.

6. CONCLUSIONS

Analytical predictions based on Hamilton's principle are compared with experimental data for adaptive stiffening of a cantilever beam with embedded charge-driven mini-actuators. The mechanical interactions between the host and the devices are modeled using eigenstrain method. Adaptive stiffening of the adaptive beam was achieved experimentally by using constant gain, position-feedback, distributed control. This paper demonstrates the ability of eigenstrain methods to model adaptive structures containing mini-devices. This paper also illustrates that no significant stiffening can be obtained for practical ranges of electrical excitation, using charge-driven actuators.

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	E (GPa)	ν	d_{31}	d_{33}	d_{15}	ε_{11}	ε_{33}
PZT-5H	64	0.31	-274	593	741	150	130
Alplex	2.4	0.3	—	—	—	—	—

Table (I): Material Properties.

Table (II): Effect of Number of Active Devices (n) on Stiffening at Constant Gain.

n	Analytical	Experimental	%Error
0	23.35	24.4	4.3
1	23.67	25.0	5.3
2	24.23	25.3	4.2

Table (III): Effect of Feedback Gain (G) on Stiffening for Two Active Devices (n = 2).

G	Analytical	Experimental	%Error
0	23.35	24.4	4.3
5	23.48	24.5	4.2
9	23.57	24.6	4.2
20	23.82	24.8	4.0
29	24.03	25.0	3.9
33	24.12	25.1	3.9
38	24.23	25.3	4.6
41	24.30	25.5	4.7
44	24.37	25.6	4.8
50	24.50	25.8	5.0
56	24.64	25.9	4.9

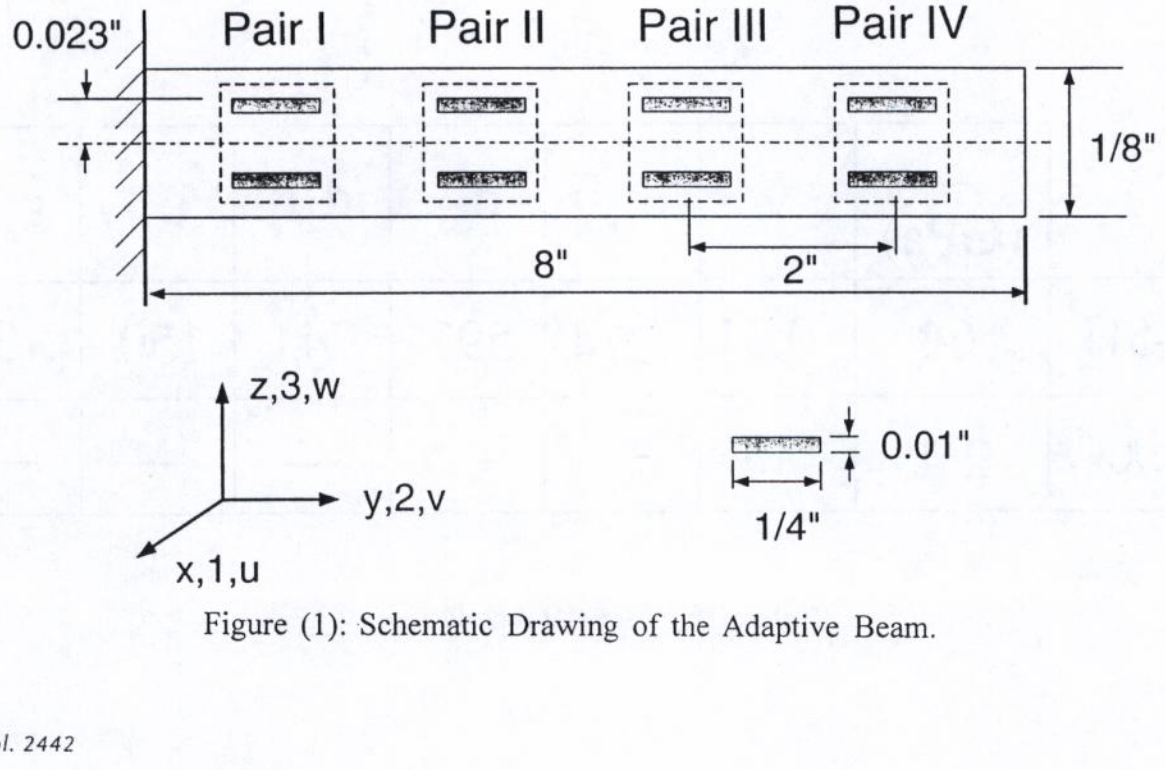

Figure (1): Schematic Drawing of the Adaptive Beam.

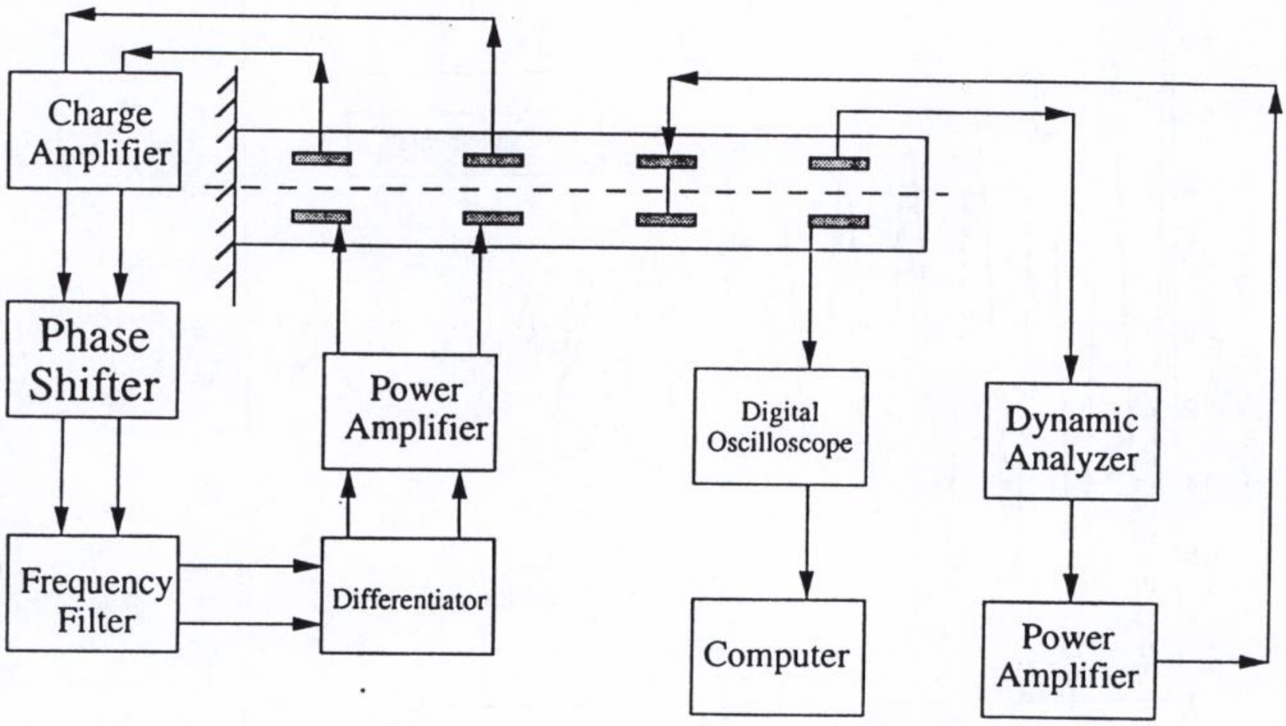

Figure (2): Experimental Setup.

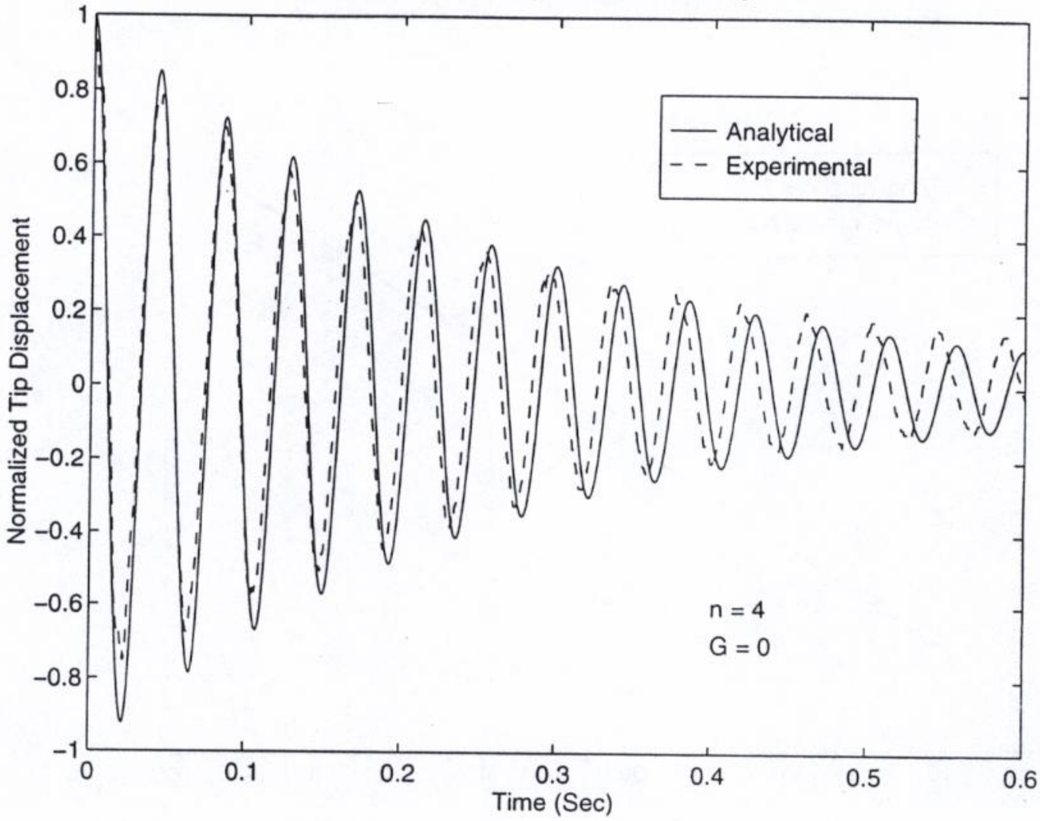

Figure (3): Adaptive Stiffening in Time Domain (Passive).

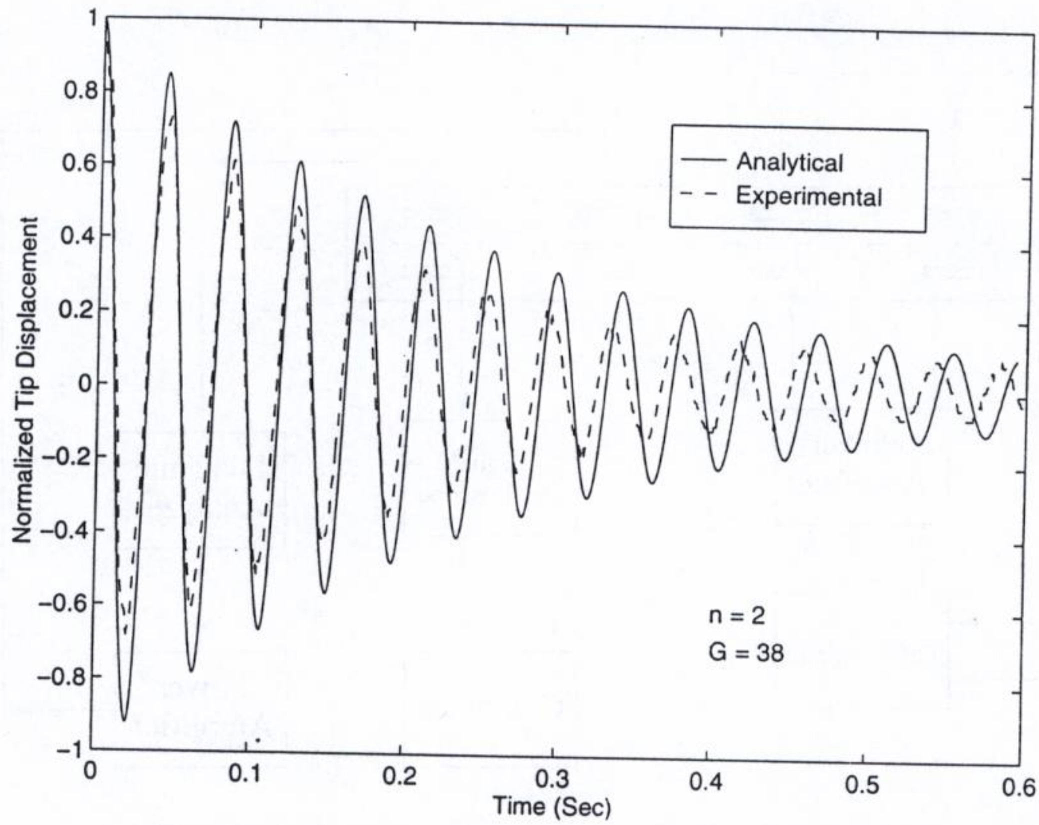

Figure (4): Adaptive Stiffening in Time Domain (Active).

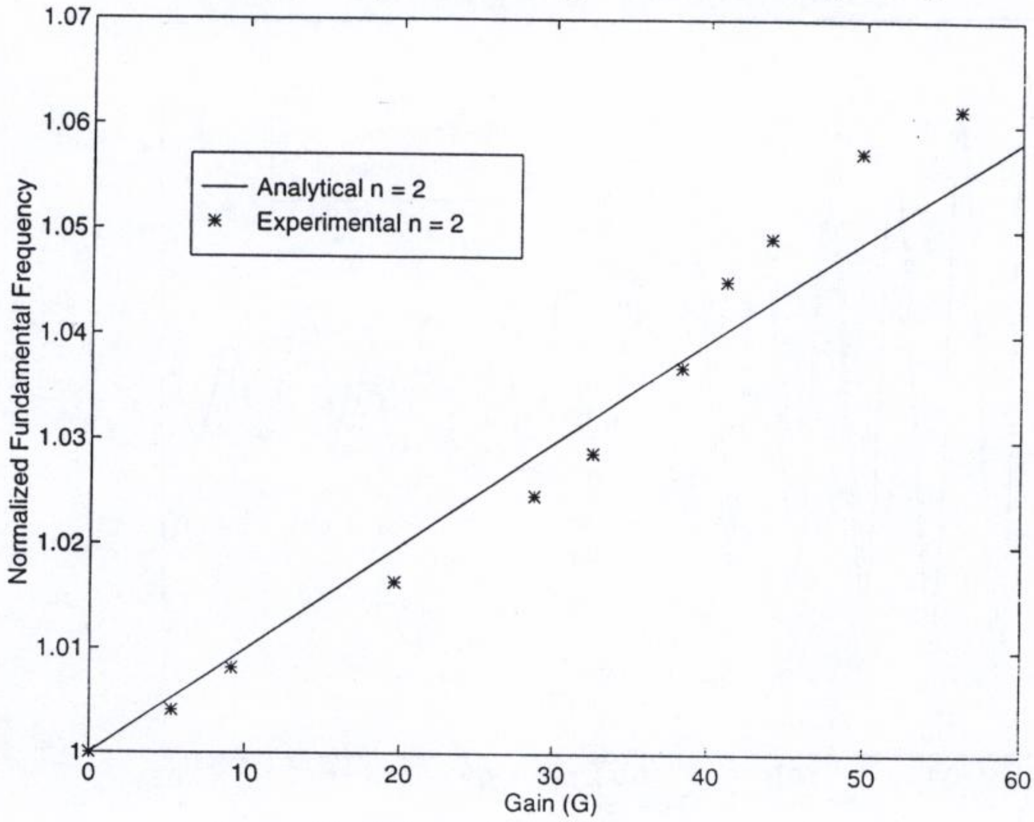

Figure (5): Effect of Feedback Gain on Stiffening.

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