JKAU: Sci., **Vol. 18,** pp: 81-84 (2006 A.D. / 1427 A.H.)

First Order Uniform Solutions for Systems of General Odd Nonlinearities

A.S. Al-Fhaid
Department of Mathematics, Faculty of Science,
King Abdulaziz University, Jeddah, Saudi Arabia

Abstract. In this paper, first order uniform solutions with respect to small parameter ε are established analytically for systems of general odd nonlinearities.

Introduction

The free oscillations[1] of many conservative systems having a single degree of freedom are governed by an equation of the form

$$\ddot{x}^* + f(x^*) = 0$$

where f is a nonlinear function of x^*. Here, $\ddot{x}^*$ is the acceleration of the system, where $f(x^*)$ is the restoring force. If $x^* = x_0^*$ be an equilibrium position of the system (i.e. $f(x^*) = 0$) and f is an analytical function at $x^* = x_0^*$; then it can be expanded in a Taylor series and we get a dimensionless equation of the form[2],

$$\ddot{\boldsymbol{u}} + \omega_0^2\, \boldsymbol{u} = \varepsilon \sum_{j=1} k_j \boldsymbol{u}^j$$

where ε is a dimensionless quantity, u is a dimensionless variable and ω_0 is a constant, the dot denotes the derivative with respect to the dimensionless time t.

In this paper, first order uniform solutions with respect to small parameter ε are established analytically for systems of general odd nonlinearities of the form

$$\ddot{\boldsymbol{u}} + \omega_0^2\, \boldsymbol{u} = \boldsymbol{\varepsilon}\, \boldsymbol{u}^{2l+1} \tag{1}$$

First Order Uniform Solution

In this section, an analytical first order uniform solutions of Equation (1) will be established for any possible non-negative integer values of l. To do so we shall use the method of multiple scales[3] as follows

■ Introduce the scales

$$T_0 = \boldsymbol{t} \quad ; \quad T_1 = e\,\boldsymbol{t}, \tag{2}$$

then using the chain rule, Eq. (1) to the first order could be written as

$$\frac{\partial^2 \boldsymbol{u}}{\partial T_0^2} + 2\varepsilon \frac{\partial^2 \boldsymbol{u}}{\partial T_0 \partial T_1} + \omega_0^2 \boldsymbol{u} = \varepsilon\, \boldsymbol{u}^{2l+1}. \tag{3}$$

■ Let

$$\boldsymbol{u} = \boldsymbol{u}_0\,(T_0\,,T_1) + \varepsilon\, \boldsymbol{u}_1\,(T_0\,,T_1), \tag{4}$$

in equation (3) and equate like power of ε we get

$$\frac{\partial^2 \boldsymbol{u}_0}{\partial T_0^2} + \omega_0^2\; \boldsymbol{u}_0 = 0\,, \tag{5}$$

$$\frac{\partial^2 \boldsymbol{u}_1}{\partial T_0^2} + \omega_0^2\; \boldsymbol{u}_1 = -2\frac{\partial^2 \boldsymbol{u}_1}{\partial T_0 \partial T_1} + \boldsymbol{u}_0^{2l+1}. \tag{6}$$

■ The solution of Equation (5) is

$$\boldsymbol{u}_0 = \boldsymbol{a}(T_1)\;\cos\,[\omega_0 T_0 + \beta(T_1)]\,, \tag{7}$$

then

$$\frac{\partial^2 \boldsymbol{u}_0}{\partial T_1 \partial T_0} = -\boldsymbol{a}\omega_0 \frac{\partial \beta}{\partial T_1}\;\cos(\omega_0 T_0 + \beta) - \omega_0 \frac{\partial \boldsymbol{a}}{\partial T_1}\sin(\omega_0 T_0 + \beta)$$

and Equation (6) becomes

$$\frac{\partial^2 \boldsymbol{u}_1}{\partial T_0^2} + \omega_0^2 \boldsymbol{u}_1 = 2\boldsymbol{a}\omega_0 \frac{\partial \beta}{\partial T_1}\cos(\omega_0 T_0 + \beta) + 2\omega_0 \frac{\partial \boldsymbol{a}}{\partial T_1}\sin(\omega_0 T_0 + \beta) +$$

$$+\frac{(2l+1)!}{2^{2l}}\boldsymbol{a}^{2l+1}\left\{\frac{1}{(l+1)!\,l!}\cos(\omega_0 T_0 + \beta) + \sum_{n=1}^{l}\frac{1}{(l+n-1)!(l-n)!}\cos\left[(2n+1)(\omega_0 T_0 + \beta)\right]\right\}$$

■ Eliminating the mixed secular terms in the above equation yields

$$2\omega_0 \frac{\partial \boldsymbol{a}}{\partial T_1} = 0\,, \tag{8}$$

$$2\boldsymbol{a}\omega_0 \frac{\partial \beta}{\partial T_1} + \frac{(2l+1)!}{2^{2l}(l+1)!\,l!}\boldsymbol{a}^{2l+1} = 0. \tag{9}$$

■ From Equation (8) it follows that ***a*** is constant and Equation (9) yields

$$\frac{\partial\beta}{\partial T_1} = -\frac{1}{2w_0}(\frac{a}{2})^{2l}\binom{2l+1}{l}$$

since $T_1 = \varepsilon\, t$, then

$$\beta = -\frac{\varepsilon}{2\omega_0}(\frac{a}{2})^{2l}\binom{2l+1}{l}t + \beta_0 \ ,$$

where β_0 is constant.

Finally, the required first order uniform solutions of the generalized Equation (1) are

$$u = a\ \cos[\{\omega_0 - \frac{\varepsilon}{2\omega_0}(\frac{a}{2})^{2l}\binom{2l+1}{l}\}t + \beta_0]$$

References

[1] **Shivamoggi, K.B.,** *Perurbation Methods for Differential Equations*, Birkhäuser, Berlin (2003).

[2] **Hinch, E.J.,** *Perurbation Methods,* Cambridge University Press, Cambridge, UK (1991).

[3] **Nayfeeh, A.H.,** *Introduction of Perurbation Techniques,* John Wiley and Sons, New York (1973).

حلول الرتبة الأولى المنتظمة لنظم غير خطية من درجة فردية عامة

عبدالرحمن صالح الفهيد
قسم الرياضيات ، كلية العلوم ، جامعة الملك عبد العزيز
جـــدة – المملكة العربية السعودية

المستخلص. تم في هذا البحث تشييد الحلول التحليلية المنتظمة من الرتبة الأولى بالنسبة إلى بارامتر صغير ، وذلك لنظم غير خطية من أي رتبة فردية.